Thermal modeling of telescope mirrors

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What exactly happens while a mirror is cooling down? It will seem overcorrected, which can be attributed to edge zones cooling faster than the center, and hence the center being slightly thicker. Also, air currents due to heat transport will affect the mirror performance. This article analyzes these thermal effects numerically, using mathematical models. Some conclusions are consistent with gut feeling, but the model is supposed to quantify the effects much better.

To start with, it is important to understand how a mirror cools and which are the parameters are important. Analytical methods can be used for rough approximation, but reality is better approached by solving the heatflow differential equations. For this a free PDE-solver tool is used: *FlexPDE*.

Models

The infinite plate approximation

The mirror can be modelled as an infinite plate of glass (without edges) to analyze only the transverse effects. The situation is comparable with a mirror where the edge is thermally isolated, in other words, there is only heat loss through front and back surfaces.

In this model, the glass surface temperature T_s is considered to be constant, while the core temperature T_0 has a certain (higher) start value. The development of the thermal distribution is therefore completely determined by the material properties of the glass plate itself.

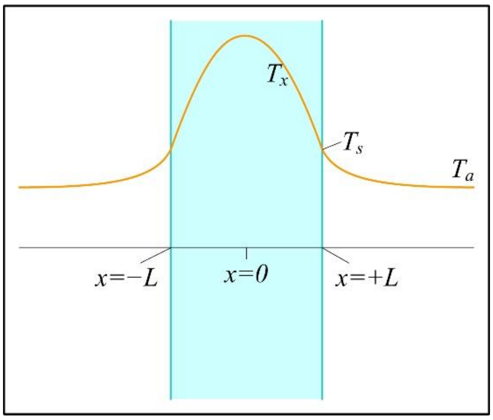

Figure 1 – Infinite plate approximation

The figure shows a plate with thickness *2L* (*-L<x<L*). The glass heat conductivity coefficient is λ (≈1.2), the specific mass ρ (≈2230), and the specific heat C_p (≈800). The transient temperature distribution (after a certain initial period) is given by:

$$T'(x,t) \propto e^{-\alpha t\left(\frac{\pi}{2L}\right)^2} \cdot \cos\left(x \cdot \frac{\pi}{2L}\right) \quad [1]$$

where

$\alpha = \frac{\lambda}{\rho \cdot C_p}$ and $T'(x,t) = \frac{T(x,t) - T_s}{T_0 - T_s}$

The magnitude of the temperature profile decreases exponentially in time, while after some time a cosine distribution forms over the thickness of the plate. The surface temperature is assumed constant at T_s and only the profile within the glass is considered.

The time behavior is described by the exponential part of the equation, which defines the peak of the cosine distribution, in other words the temperature difference between core (*x=0*) and surface (*x=L*) of the plate. This difference drops below 1% of the original value when the exponent is lower than -4.6, or effectively when the time is:

$$t \geq \frac{4.6}{\alpha \cdot \pi^2} \cdot (2L)^2 \qquad [2]$$

So apart from the material characteristics, this time limit is proportional to the squared plate thickness. For Borosilicate glass this is after approximately $0.7 \cdot d^2$ ($d=2L$, in mm), so less than 10 minutes for a 25mm plate. For Soda-Lime glass it is slightly worse: $0.9 \cdot d^2$.

The problem with this simple approximation is that the surface temperature will remain close to the core temperature when ***h*** is low, as is the case with natural air convection. A relatively warm boundary layer of air is formed on the glass surface. Forced air cooling is better, while the modeled situation would be approached best when the glass is submerged in e.g. water.

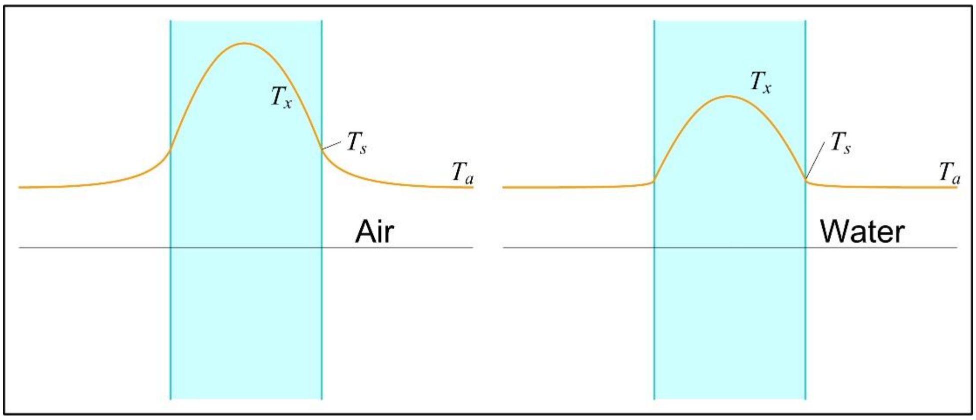

Figure 2 – Effect of environment

In reality the surrounding medium will of course be warmed up by the glass and T_s effectively changes and should be taken as a parameter. The development of the temperature in the boundary layer is related to the heat transfer coefficient ***h***, where low values represent bad and high values good transfer of heat.

$$Q = -h \cdot (T_s - T_a) \qquad [3]$$

This coefficient is the amount of heat loss per unit area and per degree temperature difference. The value of ***h*** depends on the medium the glass is suspended in, particularly the heat capacity and the amount of flow. The flow can be either forced or natural convection. The latter is visible in the telescope image because the mixing warm and cold air have different refractive coefficients.

Finally, this simple model also ignores the heat loss due to radiation.

The heat-flow equation

A more realistic result can be obtained by solving the heat flow differential equation for a representative physical model of the environment. For each location inside the glass, the heat-flow equation relates temperature distribution to the flow of heat. One can imagine that heat flows from high to low temperature areas, in order to achieve thermal equilibrium.

The advantage of this method is that a richer model of the environment can be included. For the boundaries of the area where the PDE is considered, i.e. the glass surface, so called boundary conditions are defined. Effectively these conditions state how much heat crosses the boundary, in case of the mirror this can be either by convection or by radiation. The magnitude of the boundary heat-flow is determined by the environment, for example forced air flow or natural convection and environmental temperature when regarding radiation.

The heat-flow partial differential equation (PDE) is given by:

$$\nabla\left(\lambda \cdot \frac{\partial T_x}{\partial x}\right) - \rho \cdot C_p \cdot \frac{\partial T_x}{\partial t} = 0 \qquad [4]$$

The parameter x in this case is three-dimensional vector and so is the gradient. The first term describes the heat flow on a certain moment, related to the local temperature gradient and material parameter λ. The second term describes the change of temperature on a certain location, depending on material parameters ρ and C_p. Both terms should balance out, due to the law of energy conservation.

The heat-flow PDE is solved with a PDE solver, *FlexPDE*, of which a usable demo version can downloaded for free. The mirror is modelled as a glass disk, with e certain diameter and thickness. The heat-flow equation applies to the domain enclosed by the mirror front, back and side surfaces. The transient effects however are determined by the heat loss through these surfaces: the boundary conditions, taking into account both convection and radiation. These boundary conditions are determined by the environment, like for example telescope construction and weather conditions.

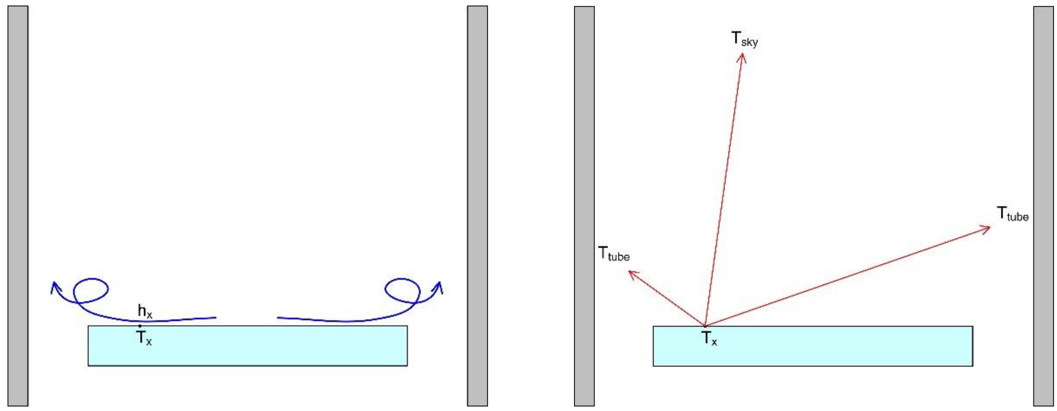

Figure 3 – Convection and radiation

In Figure 3, the convection and radiation are illustrated. When a fan is directed at the mirror center, the airflow and hence the transfer coefficient will vary over the mirror surface. The radiation is an integral of the temperature and emission coefficients of objects in all directions. This will also depend on viewpoint, i.e. location on the mirror surface.

Convection

Convective transport of heat is the transfer of heat from the glass to the surrounding air. A boundary layer of air is heated by the glass; the relatively warm air in this boundary layer flows away and is replaced by cooler ambient air. The magnitude of convective heat loss is described by equation [3].

$$Q_x = -h_x \cdot (T_x - T_a) \qquad [3]$$

Where T_a is the ambient air temperature and T_x is the temperature of the glass surface at a certain point x. The heat transfer coefficient h_x is determined by the material characteristics of the air as well as the amount of airflow. More flow means more effective transport of heat. The value of h_x therefore will be higher when a fan is used.

Typical values for an air boundary layer are:

- 1-10 for natural convection
- 10-100 for forced air flow

Note that with natural convection, a lower temperature difference (i.e. close to equilibrium) results in a lower airspeed and as a consequence h_x will also be lower. Therefore, the heat transfer coefficient is not a constant for natural convection cases!

Radiation

The second mechanism for heat transport is radiation. Radiation can cause objects to cool down even below the temperature of the surrounding air. When the surface temperature drops below the dew point, water vapor in the air will condense on the object.

Radiation is described by Stefan-Boltzmann's law:

$$\boldsymbol{Q_x = -\varepsilon_x \cdot \sigma \cdot (T_x^4 - T_e^4)} \qquad [5]$$

The glass surface temperature is given by T_x, and the temperature of the environment this surface is facing, is given by T_e. Further, σ is the Stefan-Boltzmann constant (5.67x10^{-8}) and the surface emissivity at location $\boldsymbol{x}$ is given by ε_x ($0 < \varepsilon_x < 1$), which gives the radiation efficiency compared to a black body ($\varepsilon = 1$). This is an engineering approximation called grey body emissivity, valid for a certain temperature range.

The assumption in the SB equation is that the environment is a black body. In reality this is usually not the case, and hence a grey body equivalent environment temperature T_e' must be derived first:

$$\boldsymbol{T'_e = T_e \cdot \sqrt[4]{\varepsilon_e}} \qquad [6]$$

It is interesting to know the effective temperature of the night sky, since the mirror front surface has it in full view. The universe background radiation would suggest a temperature of about 3K, but this disregards the intermediate atmosphere. In fact, when regarding thermal radiation, we are actually not looking at the universe but rather at the air itself.

An empirical model for radiation has been proposed by *Swinbank* in the sixties. His model assumes that this is a function of mainly air temperature (T_a) and humidity (Rh). Furthermore, a factor is included that takes cloud cover into account.

$$\boldsymbol{Q = (1 + C_h \cdot C_c^2) \cdot NS \cdot T_a^{5.852} \cdot Rh^{0.07195}} \qquad [7]$$

Where $NS = 8.78 \times 10^{-13}$, C_c is the degree of cloud cover [0..1] and C_h is a factor for the altitude of the cloud base (0.34, 0.18, 0.06 for low, mid and high).

From the *Swinbank* formula, the equivalent night sky temperature then is given by:

$$\boldsymbol{T_e = \sqrt[4]{\frac{Q}{\sigma}}} \qquad [8]$$

A simpler alternative approach is to directly calculate the night sky emission coefficient first. With relatively high relative humidity (Rh>50) the dew point temperature is approximated with:

$$\boldsymbol{T_d = T_a - \frac{100 - Rh}{5}} \qquad [9]$$

From this, the night sky emissivity is derived with:

$$\boldsymbol{\varepsilon_e = 0.73 + 0.0063 \cdot (T_d - 273)} \qquad [10]$$

Temperatures are in *[K]*. In most realistic cases the value of ε_e is between *0.75* and *0.80*. Using equation [6] it can be assumed as a rule of thumb that the night sky temperature is approximately ***15K*** below ambient for moderate humidity levels. Dryer (desert) air will lead to lower dew point temperatures and hence larger differences.

Simulations

The infinite plate and heat-flow models have been applied to several practical cases. The material characteristics are:

Material	ρ [kg/m³]	C_p [J/kg.K]	λ [W/m.K]	CoE [µm/m.K]	ε [..]
Plate (Soda-Lime)	2440	800	0.94	9.0	0.95
Pyrex (Borosilicate)	2230	800	1.2	3.3	0.95
Polished Aluminum	-	-	-	-	0.04 - 0.06
CVD Al + SiO overcoat	-	-	-	-	0.04

The mirrors that have been used are all 250mm diameter. Three different mirrors have been modelled, varying in thickness and material:

1. 25mm Borosilicate
2. 25mm Soda-Lime
3. 18mm Soda-Lime

As a starting point the infinite plate approximation gives the best-case cooling times:

Mirror	$T-T_a<0.1K$
1. 25mm BoroSilicate	7 min 30 sec
2. 25mm Soda-Lime	9 min 20 sec
3. 18mm Soda-Lime	4 min 50 sec

This appears to be very optimistic; as stated before the heat exchange from glass to air is far from the presumed ideal.

Static environment

The first simulations assume a static environment, i.e. the context of the mirror remains the same throughout the run. The initial glass temperature is 293K (room temperature) while the outside ambient air temperature is 283K. Note that the equivalent night sky temperature in this case will be approximately 268K, i.e. well below freezing.

Two cases are compared, natural convection airflow ($h=5\ W/K{\cdot}m^2$) and forced airflow ($h=50\ W/K{\cdot}m^2$). The edge of the mirror is assumed to be isolated, so heat loss is limited ($h=0.01\ W/K{\cdot}m^2$).

Solving the differential equations with *FlexPDE* shows a different picture. Below the ***T(t)*** graphs for the three mirrors. The simulation run is made over two hours (7,200 seconds) where it can be concluded that none of the mirrors reaches thermal equilibrium, even considering that the ambient temperature is assumed constant. Borosilicate cools slightly better than Soda-Lime glass, and of course the reduced thickness of mirror 3 clearly pays off.

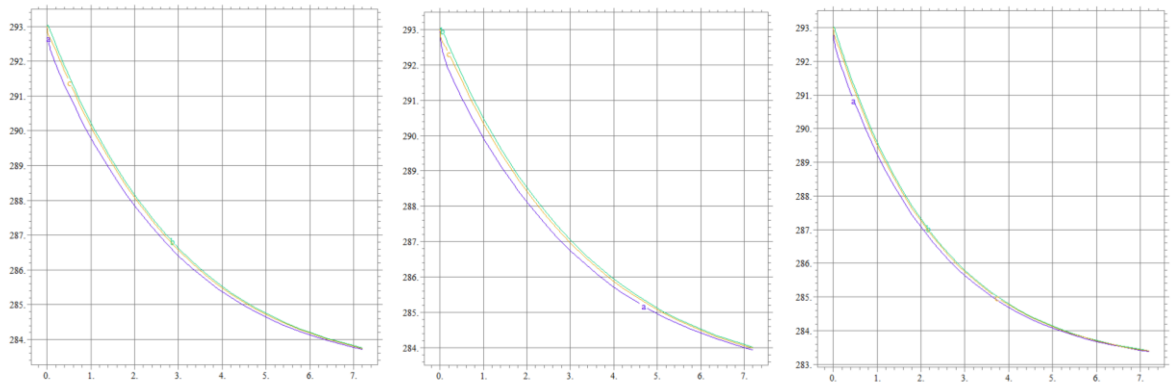

Figure 4 – Natural convection cooling of mirrors 1, 2 and 3

The second simulation run uses forced airflow on front as well as back side. One can question how realistic this is, but the two-hour simulation run shows significantly more rapid cooldown, equilibrium reached after roughly half an hour, the thinner mirror (3) again being is the quickest.

Even these simulations are optimistic, since the heat transfer coefficient is taken a constant 5 $W/K{\cdot}m^2$, while in reality it will be lower when temperature difference decreases. Also, in this case the descending ambient temperature is not taken into account. The heat loss due to emission will eventually be higher than convective heat loss.

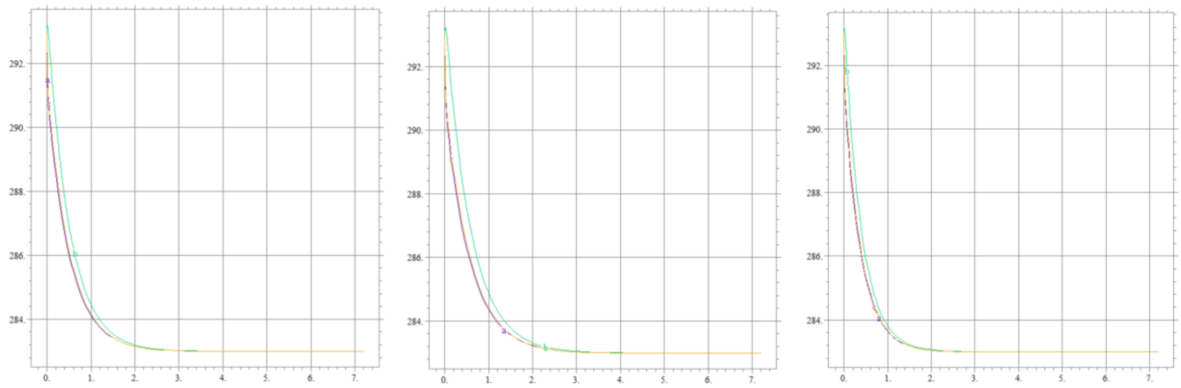

Figure 5 – Forced air cooling of mirrors 1, 2 and 3

The same behavior, but everything takes a factor 7 less time. Also note that the final temperature is below ambient, this is caused by the additional emissive cooling.

Now let's have a look at the temperature gradient inside mirror 1, using forced air cooling. Note that the gradient is mainly oriented axially, while the temperature difference after 30 minutes is 0.1K. After an hour it has become negligible.

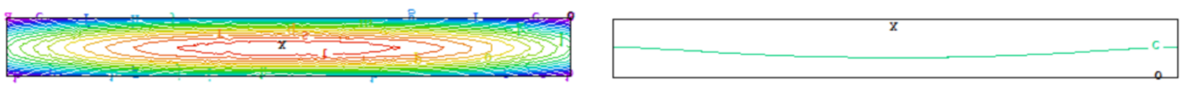

Figure 6 – Temperature gradient in mirror 1, after 30 and 60 minutes.

When the edge isolation is removed, a much higher lateral gradient is a result. The temperature difference after 30 minutes is now 0.3K. Because the gradient is mainly lateral, it will be visible as under correction.

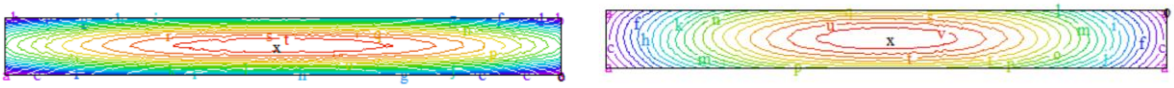

Figure 7 – Temperature gradient in mirror 1, with/without edge isolation.

Dynamic environment

Now let us assume a descending environment temperature, 5K over the 2-hour simulation (288 → 283), all simulated with mirror 1.

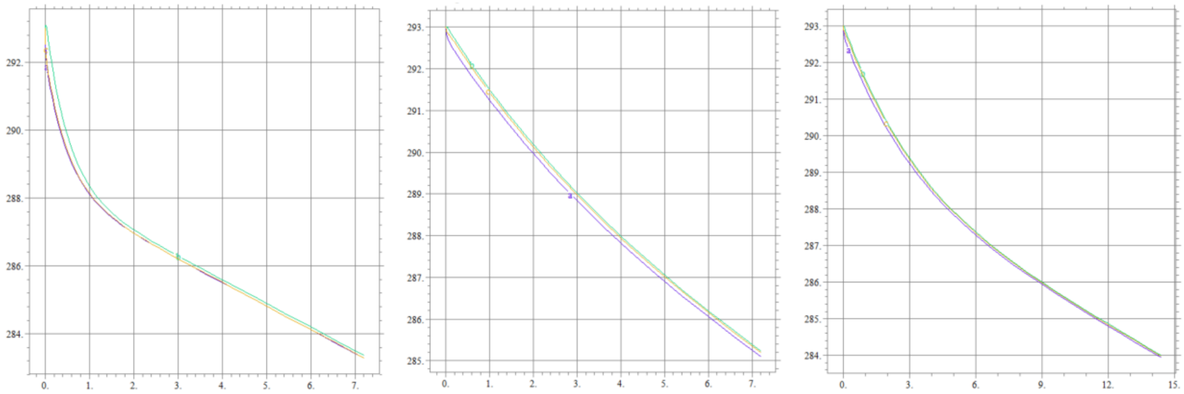

Figure 8 – Temperature in mirror 1, in dynamic environment.

The first image shows the case with forced air cooling, where the mirror easily follows the dropping ambient temperature. The middle image shows the same simulation, but with natural convection cooling. The final temperature is never reached. This is also the case when the time interval is stretched to 4 hours, the graph follows the ambient temperature but never reaches it; it is always 1K behind. This difference is too much for obtaining an aberration free image, not because of glass deformation but rather because of the disturbing boundary layer.

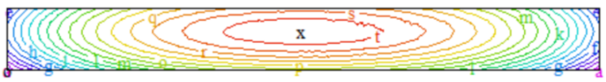

Figure 9 – Temperature gradient in mirror 1, in dynamic environment.

Figure 9 shows the temperature gradient in the mirror after 4 hours, when a stable situation is reached. The maximum temperature difference is 0.2K.

Conclusions

The physical model used in the solution of the heat flow PDE is quite crude. The radiation should be elaborated a bit more, and this is also the case for the model of convective heat loss.

For radiation, on each point of the mirror a space integral must be calculated over what radiators are visible over the half sphere in view. On the other hand, it can be concluded from the simulations that the radiative heat loss at the front surface is fairly low anyway. This is due to the low emissivity of the aluminized surface. The back of the mirror faces surfaces which approximately have ambient temperature, so this does not contribute a lot as well.

Note that the back of the secondary is a different case, having a relatively high emissivity and facing the night sky! It is important to pay some attention to the secondary and also the spider in Newton telescopes. It will help to extend the tube, to isolate the sky facing parts and coat with high reflectivity material.

The convection coefficient should be made location dependent over the entire surface area, it will for example be higher in the direct flow of the fan, but lower in more remote areas.

Still a few conclusions can be distilled from this investigation:

- Forced air cooling is required to obtain workable cooling times and a minimum of residual temperature differences.
- In the initial phase, relatively high temperature gradients are present inside the glass.
- Radiation can in principle cause a residual gradient, but the effect is relatively small.
- Cooling times are roughly proportional to the square of the glass thickness. In case of forced air flow, a safe rule of thumb is $t_c \approx 5 \cdot d^2$, where t_c is in seconds and d is thickness in mm.
- Isolation of mirror edge gives a more regular (mainly axial) temperature gradient, also during cooldown, and probably less deformation.